TRINITY COLLEGE

Year 12 Mathematics Specialist 3/4
Test 6 2022
Weighting 6%

Calculator Assumed

Simple Harmonic Motion and Statistical Inference

STUDENT'S NAME _____

DATE: Thursday 8 September **TIME:** 50 minutes **MARKS**: 50

INSTRUCTIONS:

Standard Items:	Pens, pencils, drawing templates, eraser
Special Items:	Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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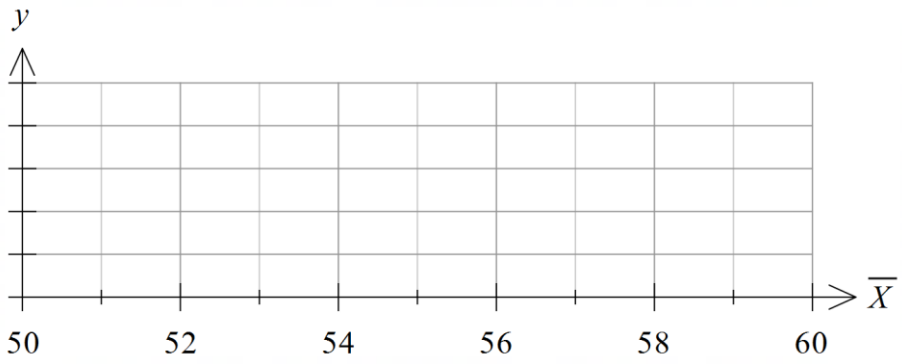
1. (10 marks)

Mr. Presser charges his phone each day and keeps accurate logs of his phone charging time. The time taken to charge the phone is normally distributed with mean $\mu = 55$ minutes and standard deviation $\sigma = 7$ minutes.

Mr. Presser randomly samples 64 phone charging times. Let $\bar{X}$ be the distribution of sample mean phone charging times for samples of size 64.

(a) Describe the distribution of $\bar{X}$. [3]

(b) Sketch the likely distribution of $\bar{X}$. [2]



(c) Describe the change of the shape of distribution $\bar{X}$ if:

(i) the sample size was to increase. [1]

(ii) the number of samples was to increase. [1]

(d) Determine the probability that the total charge time is less than 55 hours. [3]

2. (9 marks)

The velocity-displacement equation of a body is $v^2 = \pi^2(9 - x^2)$.

(a) Without using trigonometric functions, show that the body is undergoing simple harmonic motion. [3]

(b) Determine the

(i) period of the motion. [1]

(ii) maximum acceleration of the body. [2]

(iii) least time taken to move between the two points $x = 0$ and $x = 1.5$ [3]

3. (14 marks)

The movement of a particle is modelled in terms of x, the displacement in cm from the origin, and t, time in seconds.

Given $\frac{d^2x}{dt^2} = -9x$, and that our particle was initially observed at the origin with a negative velocity and travels 15 cm in one cycle:

(a) Express x in terms of t. [3]

(b) Calculate when the particle is first 1 cm away from the origin. [1]

(c) Calculate when the particle has travelled a total distance of 4 cm. [2]

(d) Calculate how far the particle has travelled from $t = 0.4$ to $t = 0.85$, and hence the average speed over this time. [2]

(e) Calculate the displacement of the particle when it first has an increasing speed of 9.87 cm/s. [3]

(f) Calculate the percentage of time the particle spends moving slower than 5 cm/s. [3]

4. (17 marks)

A first sample of 50 pizzas has the weight of cheese recorded with a sample mean of 175.0 grams and a sample standard deviation of 13.4 grams.

(a) Based on the first sample, calculate the 95% confidence interval for the mean weight of cheese on a pizza. [3]

A second sample of 150 pizzas has the weight of cheese recorded and a 99% confidence interval is calculated. The lower limit of this interval is 167 grams, and the width of the interval is 6.3 grams.

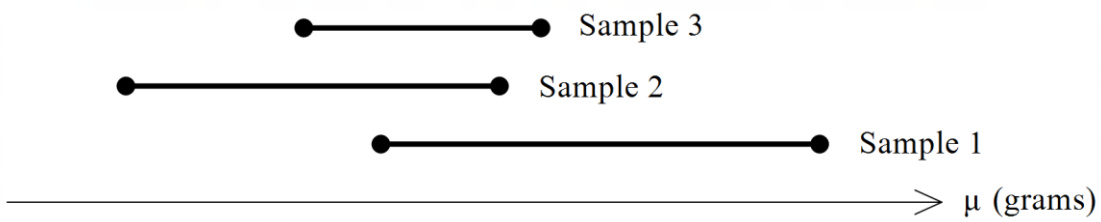
(b) Determine the sample mean for the second sample. [2]

(c) Calculate, correct to 0.1 grams, the sample standard deviation for the sample of 150 pizzas. [3]

A third sample of n pizzas has the weight of cheese recorded and has a sample standard deviation of 13.8 grams.

(d) If the probability for the mean amount of cheese used differs from μ by less than 2 grams is 96%, calculate n, the number of pizzas that need to have their cheese weighed. [4]

The confidence intervals for each sample is shown below.



(e) A student claims that "*Sample 1 has a larger sample standard deviation than Sample 2 because the confidence interval is wider"*. Comment on the validity of this claim. [3]

(f) Which confidence interval is most likely to contain the value for μ? [2]